TRIGONOMETRY

TRIGONOMETRICAL RATIOS

Angle: An angle is the amount of rotation of a revolving line with respect to a fixed line. If the rotation is in anti-clockwise direction then angle measured is positive and if it is in clock-wise direction, the angle measured is negative.

There are three systems of measuring an angle viz.

(1) Sexagesimal system or English system (degree)
(2) Circular system (radian)
(3) French system or centesimal system (grade)

First two of these three systems are commonly in use.

(1) In **Sexagesimal system**, a right angle is divided into 90 equal parts each called degree. Further, each degree is divided into sixty equal parts called minutes and each minute is divided into sixty equal parts called seconds.

Thus 1 right angle = 90 degrees (90^0)
1 degree (1^0) = 60 minutes (60')
1 minute (1') = 60 seconds (60")

(2) In **circular system** the unit of measurement is radian. One radian is the angle made by an arc of length equal to radius of a given circle at its centre.

Relation between degree and radian: If D is the degree measure of an angle and R is its measure in radians then 90 D = $\pi/2$ R

$\therefore$ **1 radian = $180/\pi$ degrees = 57^0 17' 45"** (approximately)

And **1 degree = $\pi/180$ radian**.

(3) In **French or Centesimal system**

1 right angle = 100 grades (100^g)
1 grade (1^g) = 100 minutes = (100`)
1 minute (1`) = 100 seconds = (100")

Note: Relation between the three systems	$180° = \pi$ radians = 200^g

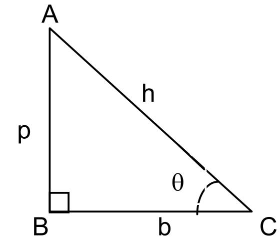
BASIC TRIGONOMETRIC RATIOS:

In a Right Triangle ABC, if θ be the angle between AC & BC we define

Sine θ or **Sin θ** = $\frac{p}{h}$

Cosine θ or **Cos θ** = $\frac{b}{h}$

Tangent θ or **Tan θ** = $\frac{p}{b}$



where p is perpendicular, h is hypotenuse, b is base of triangle

The three ratios, viz. sin θ, cos θ & tan θ are called basic trigonometric ratios.

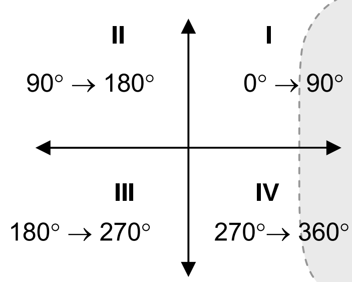
Note:

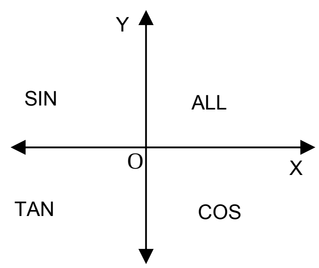
- $\frac{1}{\sin\theta}$ = **cosec** $\theta = \frac{h}{p}$
- $\frac{1}{\cos\theta}$ = **sec** $\theta = \frac{h}{b}$
- $\frac{1}{\tan\theta}$ = **cot** $\theta = \frac{b}{p}$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$ & $\cot\theta = \frac{\cos\theta}{\sin\theta}$
- $\mathbf{\sin^2\theta + \cos^2\theta = 1}$

  Proof: $(\sin\theta)^2 + (\cos\theta)^2 = \left(\frac{p}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{p^2+b^2}{h^2} = \frac{h^2}{h^2} = 1$ (Using Pythagoras result)
- $\mathbf{1 + \tan^2\theta = \sec^2\theta}$
- $\mathbf{1 + \cot^2\theta = cosec^2\theta}$

SIGN OF TRIGONOMETRIC RATIOS:

We divide the angle at a point (i.e. 360°) into 4 parts called quadrants.





In the 1st quadrant - ***all*** trigonometric ratios are positive.

2nd quadrant - only ***sin*** and ***cosec*** are positive.

3rd quadrant - only ***tan*** and ***cot*** are positive.

4th quadrant - only ***cos*** and ***sec*** are positive.

Ex. **If** $\cos A = \frac{21}{29}$ **and A lies in the fourth quadrant, find sin A and tan (A)**

Sol. cos A = 21/29

We have $\sin^2 A + \cos^2 A = 1$.

$\therefore \quad \sin^2 A = 1 - \cos^2 A = 1 - \left(\frac{21}{29}\right)^2 = 1 - \frac{441}{841} = \frac{400}{841}$

$\therefore \quad \sin A = \pm\frac{20}{29}$

$\Rightarrow \quad \sin A = -\frac{20}{29}$ ($\because$ sin A is – ve in IV quadrant)

Also, $\tan A = \frac{\sin A}{\cos A} = \frac{-20/29}{21/29} = \frac{-20}{21}$

SOME MORE RESULTS:

1. $\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$
   $\tan(-\theta) = -\tan\theta$, $\cot(-\theta) = -\cot\theta$,
   $\sec(-\theta) = \sec\theta$, $\text{cosec}(-\theta) = -\text{cosec}\,\theta$.

2. $\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$,
   $\tan(90° - \theta) = \cot\theta$, $\sec(90° - \theta) = \text{cosec}\,\theta$,
   $\cot(90° - \theta) = \tan\theta$, $\text{cosec}(90° - \theta) = \sec\theta$.

3. $\sin(180° - \theta) = \sin\theta$, $\cos(180° - \theta) = -\cos\theta$,
   $\tan(180° - \theta) = -\tan\theta$, $\cot(180° - \theta) = -\cot\theta$,
   $\sec(180° - \theta) = -\sec\theta$, $\text{cosec}(180° - \theta) = \text{cosec}\,\theta$

4. $\sin(180° + \theta) = -\sin\theta$, $\cos(180° + \theta) = -\cos\theta$,
   $\tan(180° + \theta) = \tan\theta$ $\cot(180° + \theta) = \cot\theta$,
   $\sec(180° + \theta) = -\sec\theta$, $\text{cosec}(180° + \theta = -\text{cosec}\,\theta$.

5. $\sin(270° - \theta) = -\cos\theta$, $\cos(270° - \theta) = -\sin\theta$,
   $\tan(270° - \theta) = \cot\theta$, $\sec(270° - \theta) = -\text{cosec}\,\theta$,
   $\cot(270° - \theta) = \tan\theta$, $\text{cosec}(270° - \theta) = -\sec\theta$.

6. $\sin(270° + \theta) = -\cos\theta$, $\cos(270° + \theta) = \sin\theta$,
   $\tan(270° + \theta) = -\cot\theta$, $\sec(270° + \theta) = \text{cosec}\,\theta$,
   $\cot(270° + \theta) = -\tan\theta$, $\text{cosec}(270° + \theta) = -\sec\theta$.

7. $\sin(360° - \theta) = -\sin\theta$, $\cos(360° - \theta) = \cos\theta$,
   $\tan(360° - \theta) = -\tan\theta$, $\sec(360° - \theta) = \sec\theta$,
   $\cot(360° - \theta) = -\cot\theta$, $\text{cosec}(360° - \theta) = -\text{cosec}\,\theta$.

8. $\sin(360° + \theta) = \sin\theta$, $\cos(360° + \theta) = \cos\theta$,
   $\tan(360° + \theta) = \tan\theta$, $\sec(360° + \theta) = \sec\theta$,
   $\cot(360° + \theta) = \cot\theta$, $\text{cosec}(360° + \theta) = \text{cosec}\,\theta$.

NOTE:

$(90° - \theta)$ means I quadrant
$(180° - \theta)$ means II quadrant
$(180° + \theta)$ means III quadrant
$(360° - \theta)$ means IV quadrant

You can very well see that in the first quadrant all the positive and so on.

How to remember the results?

- At 90 /270 ± θ   sin ↔ cos, tan ↔ cot , sec ↔ cosec
  Sign of result according to the quadrant in which it lies.
- At 180/360 ± θ   No change

NOTE: You should memorize the values of trigonometric ratios of 0°, 30°, 45°, 60°, 90°, so that you can derive the value of other ratios when required. Here the values are given in a tabular form.

θ	0°	30°	45°	60°	90
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	∞

NOTE: *To remember this table, just remember the values for sin θ. Then reverse all the values and get the corresponding ones for cos θ. Then tan θ can be directly obtained by dividing sin θ by cos θ.*

Ex. **Evaluate: tan 120^0 and cosec (-585^0)**

Sol. (i) $\tan 120^0 = \tan(90^0 + 30^0) = -\cot 30^0 = -\sqrt{3}$.

Alternatively, we can write

$\text{Tan } 120^0 = \tan(180^0 - 60^0) = -\tan 60^0 = -\sqrt{3}$.

(ii) $\text{cosec}(-585^0) = -\text{cosec } 585^0$ [∵ $\text{cosec}(-\theta) = -\text{cosec }\theta$, for all θ]

$= -\text{cosec}(360^0 + 225^0) = -\text{cosec } 225^0 = -\text{cosec}(180^0 + 45^0) = -(-\text{cosec } 45^0)$

[∵ $\text{cosec}(180^0 + \theta) = -\text{cosec }\theta$] = $1/\sqrt{2}$

Ex. **Simplify** $\dfrac{\tan(90^0+\theta)\sin(180^0+\theta)\sec(270^0+\theta)}{\cos(270^0-\theta)\text{cosec}(180^0-\theta)\cot(360^0-\theta)}$.

Sol. $\tan(90^0 + \theta) = -\cot\theta$, $\sin(180^0 + \theta) = -\sin\theta$

$\text{Sec}(270^0 + \theta) = \text{cosec }\theta$, $\cos(270^0 - \theta) = -\sin\theta$

$\text{Cosec}(180^0 - \theta) = \text{cosec }\theta$, $\cot(360^0 - \theta) = -\cot\theta$

∴ Given expression = $\dfrac{(-\cot\theta)(-\sin\theta)(\text{cosec }\theta)}{(-\sin\theta)(\text{cosec }\theta)(-\cot\theta)} = 1$.

DOMAIN and RANGE of trigonometrical functions

	Domain	**Range**
Sin A	R	[– 1, 1]
cos A	R	[– 1, 1]
tan A	R – {(2n + 1) π/2, n ∈ z}	(– ∞ , ∞) = R
Cosec A	R – {(nπ , n ∈ z}	(– ∞ , – 1] ∪ [1, ∞)
sec A	R – {2n + 1) π/2, n ∈ z}	(– ∞ , – 1] ∪ [1, ∞)
cot A	R – {nπ, n ∈ z}	(– ∞ , ∞) = R

Thus, |sin A| ≤ 1, |cos A| ≤ 1, |sec A| ≥ 1 and |cosec A| ≥ 1

Ex. If x and y be real, show that the equation $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible only when x = y.

Sol. We know that sec θ is numerically ≥ 1

$\therefore \quad \sec^2\theta \geq 1$

$\therefore \quad \frac{4xy}{(x+y)^2} \geq 1 \qquad \left(\because \sec^2\theta = \frac{4xy}{(x+y)^2}\right)$

$\therefore \quad 4xy \geq (x+y)^2 \qquad (\because (x+y)^2 \text{ is } +\text{ve})$

$\therefore \quad (x+y)^2 - 4xy \leq 0$

$\therefore \quad (x-y)^2 \leq 0$ (Note this step)

$\therefore \quad (x-y)^2 = 0$ [∵ Square of a real number cannot be – ve]

$\Rightarrow \quad x - y = 0 \Rightarrow x = y.$

$\therefore$ The equation $\sec^2\theta = \frac{4xy}{(x+y)^2}$ is possible only when x = y.

IMPORTANT FORMULAE

Sum and Difference formulae

- **sin (A + B) = sin A cos B + cos A sin B**
- **sin (A – B) = sin A cos B – cos A sin B**
- **cos (A + B) = cos A cos B – sin A sin B**
- **cos (A – B) = cos A cos B + sin A sin B**
- **tan(A + B) =** $\frac{\tan A + \tan B}{1 - \tan A \tan B}$ **tan (A – B) =** $\frac{\tan A - \tan B}{1 + \tan A \tan B}$

NOTE: $\tan(45° + A) = \frac{1+\tan A}{1-\tan A}$ $\tan(45° - A) = \frac{1-\tan A}{1+\tan A}$

- **sin (A + B) sin (A – B) = $\sin^2 A - \sin^2 B$**
- **cos (A + B) cos (A – B) = $\cos^2 A - \sin^2 B$**
- **tan (A + B + C) =** $\frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$
- **Value of sin 15° & cos 15°**

  $\sin 15° = \cos 75° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$

  $\cos 15° = \sin 75° = \frac{\sqrt{3} + 1}{2\sqrt{2}}$

Formulae for the transformation of a product of two circular functions into algebraic sum of two circular functions and vice-versa.

A – B formulae: (product into sum conversion)

- **2 sin A cos B = sin (A + B) + sin (A – B)**
- **2 cos A sin B = sin (A + B) – sin (A – B)**
- **2 cos A cos B = cos (A + B) + cos (A – B)**
- **2 sin A sin B = cos (A – B) – cos (A + B)**

C – D formulae (sum into product)

- **sin C + sin D = 2 sin** $\frac{C+D}{2}$ **cos** $\frac{C-D}{2}$
- **sin C – sin D = 2 cos** $\frac{C+D}{2}$ **sin** $\frac{C-D}{2}$
- **cos C + cos D = 2 cos** $\frac{C+D}{2}$ **cos** $\frac{C-D}{2}$
- **cos C – cos D = 2 sin** $\frac{C+D}{2}$ **sin** $\frac{D-C}{2}$

The above formulae are known as **'C, D' formulae.**

Note: Tip to remember the A – B and C – D formulae

sin cos ⇔ sin + sin
cos sin ⇔ sin – sin
cos cos ⇔ cos + cos
sin sin ⇔ cos – cos

Ex. **(i)** $2 \sin 3\theta \cos 5\theta$
$= \sin(3\theta + 5\theta) + \sin(3\theta - 5\theta)$
[$\because$ $2 \sin A \cos B = \sin(A + B) + \sin(A - B)$.
Here $A = 3\theta$ and $B = 5\theta$]
$= \sin 8\theta + \sin(-2\theta)$
$= \sin 8\theta - \sin 2\theta$.

(ii) $\cos 9\theta \sin 2\theta = \frac{1}{2}[2 \cos 9\theta \sin 2\theta]$
$= \frac{1}{2}[\sin(9\theta + 2\theta) - \sin(9\theta - 2\theta)]$
$= \frac{1}{2}[\sin 11\theta - \sin 7\theta]$

Formulae for t-ratios of multiple and sub-multiple angles

1. $\sin 2\theta = 2 \sin\theta \cos\theta$ ***or*** $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$$= \frac{2\tan\theta}{1+\tan^2\theta} \qquad = \frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

2. $\cos 2\theta = \begin{cases} 1-2\sin^2\theta \\ \cos^2\theta-\sin^2\theta \\ 2\cos^2\theta-1 \end{cases}$ ***or*** $\cos\theta = \begin{cases} 1-2\sin^2\frac{\theta}{2} \\ \cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2} \\ 2\cos^2\frac{\theta}{2}-1 \end{cases}$

$$= \frac{1-\tan^2\theta}{1+\tan^2\theta} \qquad = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

3. $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$ and $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$

4. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ or $\tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

5. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

6. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

7. $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

8. $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$, where $\theta \neq (2n+1)\pi$

9. $\frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$, where $\theta \neq (2n)\pi$

10. $\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{\theta}{2}$, where $\theta \neq (2n+1)\pi$

11. $\frac{1+\cos\theta}{1-\cos\theta} = \cot^2\frac{\theta}{2}$, where $\theta \neq 2n\pi$

12. $\cos A \cos 2A \cos 2^2 A \ldots \cos 2^{n-1} A = \frac{\sin 2^n A}{2^n \sin A}$

Ex. If $2\cos\theta = x + \frac{1}{x}$, prove that

$2\cos 3\theta = x^3 + \frac{1}{x^3}$.

Sol. We have, $2\cos\theta = x + \frac{1}{x}$.

$2\cos 3\theta = 2(4\cos^3\theta - 3\cos\theta)$

$= 8\cos^3\theta - 6\cos\theta$

$= (2\cos\theta)^3 - 3(2\cos\theta)$

$= \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= x^3 + \frac{1}{x^3} + 3x \times \frac{1}{x}\left(x+\frac{1}{x}\right) - 3\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

Value of Some Commonly Used Angles

$\sin 9^0 = \frac{\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}}}{4} = \cos 81^0$

$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}} = \cos 75^\circ$

$\sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ$

$\sin 36° = \dfrac{\sqrt{10-2\sqrt{5}}}{4} = \cos 54^o$

$\sin 54° = \dfrac{\sqrt{5}+1}{4} = \cos 36°$

$\sin 72° = \dfrac{\sqrt{10+2\sqrt{5}}}{4} = \cos 18°$

$\sin 75° = \dfrac{\sqrt{3}+1}{2\sqrt{2}} = \cos 15°$

$\sin 81^0 = \dfrac{\sqrt{3+\sqrt{5}}+\sqrt{3-\sqrt{5}}}{4} = \cos 9^0$

$\tan\left(22\frac{1}{2}°\right) = \sqrt{2}-1$

$\tan\left(67\frac{1}{2}°\right) = \sqrt{2}+1.$

PROOF: Let $\theta = 18^0$

$\Rightarrow\ 2\theta = 90^0 - 3\theta$

$\Rightarrow\ \sin 2\theta = \sin(90^0 - 3\theta) = \cos 3\theta$

$\Rightarrow\ 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow\ \cos\theta\,(4(1-\sin^2\theta) - 3 - 2\sin\theta) = 0$

$\Rightarrow\ 4\sin^2\theta + 2\sin\theta - 1 = 0 \qquad [\because\ \cos 18^o \neq 0]$

$\Rightarrow\ \sin\theta = \dfrac{-2\pm\sqrt{4+16}}{8} = \dfrac{-1\pm\sqrt{5}}{4}$

$\Rightarrow\ \sin 18^o = \dfrac{\sqrt{5}-1}{4}$

$\cos 18^o = \sqrt{1-\sin^2 18} = \sqrt{1-\dfrac{5+1-2\sqrt{5}}{4}}$

$= \sqrt{\dfrac{10+2\sqrt{5}}{16}} = \dfrac{\sqrt{10+2\sqrt{5}}}{4}$

Now, the other t-rations of 18^0 can be easily found out. The identity $\cos 2\theta = 1 - 2\sin^2\theta$ implies

$\cos 36^o = 1 - 2\left(\dfrac{5+1-2\sqrt{5}}{16}\right) = \dfrac{\sqrt{5}+1}{4}$

Also, $\sin 36^o = \sqrt{1-\cos^2 36} = \sqrt{1-\dfrac{5+1+2\sqrt{5}}{16}} = \dfrac{\sqrt{10-2\sqrt{5}}}{4}$.

The other t-ratios of 36^0 can now be found out easily.

Note:

$\sin\theta\sin(60^0-\theta)\sin(60^0+\theta) = \frac{1}{4}\sin 3\theta$

$\tan\theta\tan(60^0-\theta)\tan(60^0+\theta) = \frac{1}{4}\tan 3\theta$

$\cos 36^0\cos 72^0 = \frac{1}{4}$

$\cos\theta\cos(60^0-\theta)\cos(60^0+\theta) = \frac{1}{4}\cos 3\theta$

$\cos 36^0 - \cos 72^0 = \frac{1}{2}$

CONDITIONAL IDENTITIES

If **A + B + C = 180°**, then

(i) **sin 2A + sin 2B + sin 2C = 4 sin A sin B sin C**

Proof: L.H.S. = (sin 2A + sin 2 B) + sin 2C

$= 2\sin\frac{2A+2B}{2}\cos\frac{2A-2B}{2} + \sin 2C$

= 2 sin (A + B) cos (A – B) + sin 2 C

= 2 sin (π – C) cos (A – B) + 2 sin C cos C

= 2 sin C [cos (A – B) + cos (π – (A + B))]

= 2 sin C [cos (A – B) – cos (A + B)]

$= 2\sin C\left[2\sin\frac{A-B+A+B}{2}\sin\frac{A-B-A+B}{2}\right]$

= 4 sin A sin B sin C = R.H.S.

(ii) **cos 2A + cos 2B + cos 2C = – 1 – 4 cos A cos B cos C**

(iii) **sin A + sin B + sin C =** $\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

(iv) **cos A + cos B + cos C** $= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

(v) **tan A + tan B + tan C = tan A tan B tan C**

(vi) $\tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

TRIGONOMETRICAL EQUATIONS

sin x = 0 ⇔ **x = nπ**, n ∈ I

cos x = 0 ⇔ **x = (2n + 1) π/2** , n ∈ I

tan x = 0 ⇔ **x = nπ**, n ∈ I

sin x = sin α ⇔ **x = nπ + (– 1)^n**α, n ∈ I

cos x = cos α ⇔ **x = 2nπ** ± α, n ∈ I

tan x = tan α ⇔ **x = nπ +** α, n ∈ I

$\sin^2 x = \sin^2 \alpha$ ⇔ **x = nπ** ± α, n ∈ I

$\cos^2 x = \cos^2 \alpha$ ⇔ **x = nπ** ± α, n ∈ I

$\tan^2 x = \tan^2 \alpha$ ⇔ **nπ** ± α, n ∈ I

Expressions of sin A/2 and cos A/2 in terms of sin A

We have $\left(\sin\frac{A}{2} + \cos\frac{A}{2}\right)^2 = 1 + \sin A$ and $\left(\sin\frac{A}{2} - \cos\frac{A}{2}\right)^2 = 1 - \sin A$

so that $\sin\frac{A}{2} + \cos\frac{A}{2} = \pm\sqrt{1+\sin A}$ and $\sin\frac{A}{2} - \cos\frac{A}{2} = \pm\sqrt{1-\sin A}$

By adding and subtracting, we have $2\sin\frac{A}{2} = \pm\sqrt{1+\sin A} \pm \sqrt{1-\sin A}$ … (I)

and $2\cos\frac{A}{2} = \mp\sqrt{1+\sin A} \mp \sqrt{1-\sin A}$ ……...(II)

$2\sin\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$ $2\cos\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$	$2\sin\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$ $2\cos\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$
$2\sin\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$ $2\cos\frac{A}{2} = \sqrt{1+\sin A} + \sqrt{1-\sin A}$	$2\sin\frac{A}{2} = -\sqrt{1+\sin A} - \sqrt{1-\sin A}$ $2\cos\frac{A}{2} = -\sqrt{1+\sin A} + \sqrt{1-\sin A}$

MAXIMUM AND MINIMUM VALUES OF TRIGONOMETRIC FUNCTIONS

Trigonometric functions of the form **y = a sin x + b cos x.**

Then, by putting $a = r\cos\theta$, $b = r\sin\theta$, we have, y = a sin x + b cos x

$y = r\cos\theta\sin x + r\sin\theta\cos x = r\sin(x+\theta)$, (where $r = \sqrt{(a^2+b^2)}$, $\tan\theta = b/(a)$)

Since, $-1 \le \sin(x+\theta) \le 1$ for all values of x.

Therefore, $-r \le r\sin(x+\theta) \le r$ for all x

$-\sqrt{(a^2+b^2)} \le y \le \sqrt{(a^2+b^2)}$ for all x.

Maximum and Minimum values of a sinx + b cosx	MAX = $\sqrt{(a^2+b^2)}$ MIN = $-\sqrt{(a^2+b^2)}$

PERIODIC FUNCTION:

A function y = f(x) is said to be *periodic* if there exists a real number (k > 0) such that f(x + k) = f(x), for all x.

If y = f(x) is a periodic function and k is the smallest positive real number such that f(x + k) = f(x), for all x, then k is called the *period* of the periodic function f.

For any angle θ, we know that

(i) $\sin(2\pi+\theta) = \sin\theta$ (ii) $\cos(2\pi+\theta) = \cos\theta$

(iii) $\tan(\pi+\theta) = \tan\theta$ (iv) $\cot(\pi+\theta) = \cot\theta$

(v) $\sec(2\pi+\theta) = \sec\theta$ (vi) $\text{cosec}(2\pi+\theta) = \text{cosec}\,\theta$

$\therefore$ The trigonometric functions are all periodic. The periods of the functions are -

sine, cosine , secant and cosecant $\to$ 2π (= 360^0) each.

tangent and cotangent are $\to \pi$(= 180^0) each.